

## Math 30-2 Unit 7 Assignment

Name:

## Logarithmic Functions

Total Marks: \_\_\_\_/33

## Multiple Choice

Identify the choice that best completes the statement or answers the question.

1. Which exponential equation correctly represents the logarithmic equation  $y = \log 50$ ?

- A.  $50^y = 10$
- ☒ B.  $10^y = 50$
- C.  $y^{50} = 10$
- D.  $y^{10} = 50$

2. Which logarithmic equation correctly represents the exponential equation  $e^x = 7$ ?

- ☒ A.  $x = \ln 7$
- B.  $x = \ln 10$
- C.  $7 = \ln x$
- D.  $10 = \ln x$

3. Evaluate the logarithmic expression  $\log_{\frac{1}{4}} \left( \frac{1}{256} \right)$ 

- A. 2                      B. 3                      ☒ C. 4                      D. 5

Solution

$$\frac{1}{256} = \left( \frac{1}{4} \right)^4$$

$$\Rightarrow \log_{\frac{1}{4}} \left( \left( \frac{1}{4} \right)^4 \right)$$

Apply log rule:  $\log_a (a^x) = x$

$$= \underline{\underline{4}}$$

$$\log_3 18 + \log_3 \left(\frac{3}{2}\right)$$

4. Evaluate:

A. 1

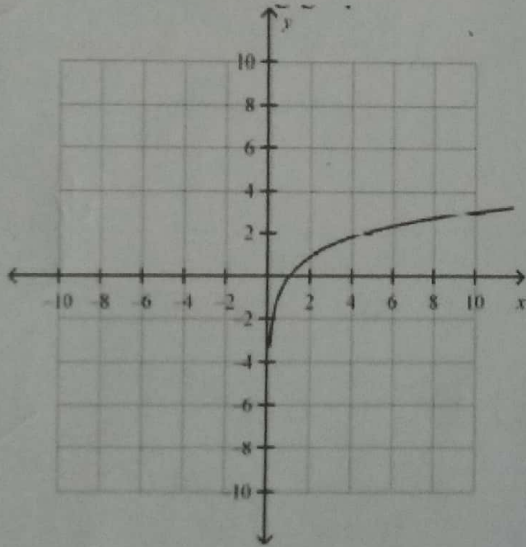
B. 2

☒ C. 3

D. 4

$$\begin{aligned} \log_3 \left( \frac{3}{2} \cdot 18 \right) & \Rightarrow 3 \log_3 (3) = \underline{\underline{3}} \\ & = \log_3 (27) \quad \text{but } \log_3 3 = 1 \\ & = \log_3 (3^3) \Rightarrow 3 \times 1 \end{aligned}$$

5. Match the following graph with its function.



A.  $y = -\frac{1}{3} \ln x$

☒ B.  $y = 3 \log x$

C.  $y = -\frac{1}{3} (3)^x$

D.  $y = 0.3(10)^x$

6. A laboratory received a shipment of 1000 g of a radioactive compound. Only 31.25 g of the compound

$$A = A_0 \left( \frac{1}{2} \right)^{\frac{t}{h}}$$

remained after 10 days. The half-life equation is , where  $A$  represents the amount of the substance remaining,  $A_0$  represents the initial amount of the substance,  $t$  represents the time, and  $h$  represents the time at which only half of the substance remains.

The half-life of the compound is

- A. 1/2 day  
B. 1 day  
✓ C. 2 days  
D. 4 days

$$A = A_0 \left( \frac{1}{2} \right)^{\frac{t}{h}}$$

$$2^{\frac{t}{h}} = 1000 / 31.25 = 32$$

$$2^{\frac{t}{h}} = 2^5$$

$$\frac{t}{h} = 5$$

$$\frac{10}{h} = 5$$

$$h = \frac{10}{5} = \underline{\underline{2 \text{ days}}}$$

7. Evaluate:  $3 \log_8 24 - 3 \log_8 3$

- ✓ A. 3  
B. 7  
C. 1  
D. 64

Solution

$$\Rightarrow \log_8 (24^3) - \log_8 (3^3)$$

$$= \log_8 \left( \frac{24^3}{3^3} \right)$$

$$= \log_8 (512)$$

$$= \log_8 (8^3)$$

$$= 3 \log_8 8$$

$$\log_8 8 = 1$$

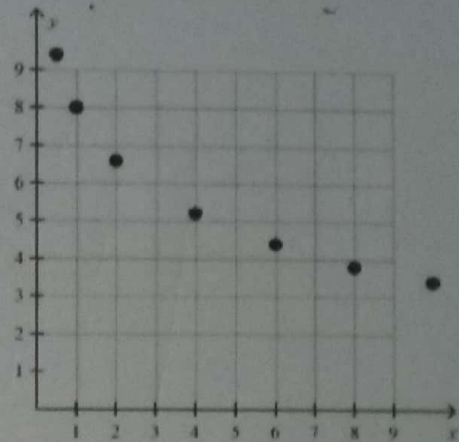
$$\Rightarrow 3 \times 1$$

$$= \underline{\underline{3}}$$

8. A scatter plot is drawn using the data set shown below. Identify the regression equation.

See answer next page

|     |     |   |     |     |     |     |     |
|-----|-----|---|-----|-----|-----|-----|-----|
| $x$ | 0.5 | 1 | 2   | 4   | 6   | 8   | 10  |
| $y$ | 9.3 | 8 | 6.8 | 5.1 | 4.3 | 3.9 | 3.5 |



- A.  $y = 9.5 - \ln x$   
 B.  $y = 9.5 - 2 \ln x$   
 C.  $y = 8.0 - \ln x$   
☒ D.  $y = 8.0 - 2 \ln x$

9. In 1935 the California seismologist Charles Richter devised a logarithmic scale that bears his name and is still widely used in the United States. The magnitude  $M$  on the Richter scale is given as follows:

$M = \frac{2}{3} \log \left( \frac{E}{E_0} \right)$ , where  $E$  is the energy released by the earthquake, measured in joules, and  $E_0$  is the energy released by a very small reference earthquake, which has been standardized to be

$$E_0 = 10^{4.40} \text{ joules}$$

The 1906 San Francisco earthquake released approximately  $5.96 \times 10^{16}$  joules of energy. What was its magnitude on the Richter scale? Compute the answer to two decimal places.

- A. -0.15  
 B. -2.41  
☒ C. 8.25  
 D. 12.38

$$M = \frac{2}{3} \log \left( \frac{E}{E_0} \right)$$

$$E_0 = 10^{4.40} \text{ Joules}$$

$$E = 5.96 \times 10^{16} \text{ Joules}$$

$$M = \frac{2}{3} \log \left( \frac{5.96 \times 10^{16}}{10^{4.40}} \right)$$

$$= \frac{2}{3} \log (5.96 \times 10^{11.6})$$

$$= \frac{2}{3} [\log(5.96) + 11.6 \log 10]$$

$$= \underline{\underline{8.25}}$$

**Written Response**

Please show/explain your work for full marks.

1. Write
- $0.2 = \ln x$
- as an exponential equation. (1 mark)

$$0.2 = \ln(x) \quad \left| \text{Therefore, } x = \underline{\underline{e^{0.2}}} \right.$$

$$\Rightarrow e^{0.2} = x$$

2. Write
- $56 = 10^y$
- as a logarithmic equation. (1 mark)

$$56 = 10^y \quad \left| \Rightarrow y = \frac{\ln(56)}{\ln(10)} \right.$$

$$\Rightarrow \ln(56) = y \ln(10) \quad \left| \therefore y = \log_{10} 56 \right.$$

3. Which expression has the greater value? (1 mark)

A.  $\log 25 - \log 20$

$$\log 25 - \log 20$$

$$= \log \left( \frac{25}{20} \right)$$

$$= \log \left( \frac{5}{4} \right)$$

B.  $\log_6 25 - \log_6 20$

$$\log_6 25 - \log_6 20$$

$$= \log_6 \left( \frac{25}{20} \right)$$

$$= \log_6 \left( \frac{5}{4} \right)$$

Comparing the two,  
it is clearly evident  
that  $\log_6 25 - \log_6 20$   
is of a greater value

4. Calculate, to the nearest tenth, the pH of a solution with a hydrogen ion concentration of
- $1.5 \times 10^{-10}$
- mol/L. Recall that pH,
- $p$
- , is defined by the equation
- $p(x) = -\log x$
- where the concentration of hydrogen ions,
- $x$
- , in a solution is measured in moles per litre. (1 mark)

$$pH = -\log(1.5 \times 10^{-10})$$

$$pH = -[\log 1.5 + \log 10^{-10}]$$

$$pH = -[\log 1.5 + -10 \log 10]$$

$$\log 1.5 = 0.176 \quad \log 10 = 10$$

$$pH = -[0.176 - (10 \times 1)]$$

$$pH = -[0.176 - 10]$$

$$pH = -(-9.824)$$

$$\underline{\underline{pH = 9.8}}$$

5. Solve the equation  $7^x = 100$  to two decimal places. (1 mark)

$$\begin{array}{l|l} 7^x = 100 & \therefore x = \frac{2 \ln 10}{\ln 7} \\ x \ln 7 = \ln 100 & \\ \Rightarrow x = \frac{\ln 100}{\ln 7} & = 2.37 \end{array}$$

6. David inherits \$10 000 and invests in a guaranteed investment certificate (GIC) that earns, 6%, compounded semi-annually. A function that models the growth of the investment is

$$y = 10000 \left( 1 + \frac{0.06}{2} \right)^x$$

, when  $x$  = number of semi-annual compounding periods and  $y$  = value of investment, in dollars.

a. Explain how you would use technology to determine how many compounding periods it will take for the investment to be worth \$11 000 at 6%, compounded semi-annually. (2 marks)

$$\begin{array}{l} 11000 = 10000(1 + 0.03)^x \\ \frac{11}{10} = (1.03)^x \\ \log\left(\frac{11}{10}\right) = x \log(1.03) \end{array} \left| \begin{array}{l} \frac{\log(1.1)}{\log(1.03)} = x \\ X = \frac{\log(1.1)}{\log(1.03)} \end{array} \right| X = 3.22$$

b. Modify the exponential function to reflect an interest rate of 6% compounded monthly, when  $x$  = number of monthly compounding periods and  $y$  = value of investment, in dollars. (1 mark)

When 6% compounded monthly when  $x$  = number of compounding period.

$$y = 10000 \left( 1 + \frac{0.06}{12} \right)^x$$

7. Rado thinks he has a proof that  $\log 25 = 1$ . Identify Rado's error and finish his solution.

(2 marks)

**Rado's Proof**

$$\log 25$$

$$= \log 5 + \log 5$$

$$= 2 \log 5$$

$$= \log 10$$

$$= 1$$

I used the product law.

I simplified the expression.

I used the power law.

I simplified the logarithm.

Errors in Rado's proof

$$2 \log 5 \neq \log 10$$

$$\text{So, } 2 \log 5 = \log 5^2 \dots \text{Power law} \\ = \log 25$$

$$\text{Hence, } \log 25 \neq 1$$

8. Solve  $7^{3x} = 10^{x+4}$ , and round your answer to one decimal place. Show your work. (2 marks)

$$7^{3x} = 10^{x+4}$$

$$\Rightarrow 3x \ln 7 = (x+4) \ln 10$$

$$\Rightarrow x = \frac{4 \ln 10}{3 \ln 7 - \ln 10}$$

$$\approx \underline{\underline{2.6}}$$

9. The population of frogs in a marsh was recorded for several years.

| Year         | 1   | 2   | 3   | 4   | 6   | 7   | 8   |
|--------------|-----|-----|-----|-----|-----|-----|-----|
| Number Frogs | 252 | 272 | 284 | 290 | 304 | 308 | 312 |

a. Determine the regression equation in the form that models the data. Round all values to the nearest hundredth. (1 mark)

$$y = bx + a$$

$$\Rightarrow y = 7.9x + 253.88$$

b. Estimate the frog population in the 5th year. Show your work. (1 mark)

$$y = (7.9 \times 5) + 253.88 \quad | \quad = \underline{\underline{293.38}}$$

c. Estimate the frog population in the 12th year. Show your work. (1 mark)

$$y = (7.9 \times 12) + 253.88 \\ = \underline{\underline{348.68}}$$

10. A scuba diver used her camera equipment to measure the intensity of light, in lux, as she dove into a lake. She compared her readings with the depth of water at each point.

|                       |     |     |     |     |     |
|-----------------------|-----|-----|-----|-----|-----|
| Light Intensity (lux) | 5.0 | 4.0 | 3.0 | 2.0 | 1.0 |
| Depth of Water (m)    | 2.3 | 3.1 | 4.0 | 5.4 | 7.7 |

a. Determine the regression equation in the form that models the data. Round all values to the nearest hundredth. (1 mark)

$$y = -1.31x + 8.43$$

b. At what depth, to the nearest tenth of a metre, is the light intensity 3.6 lux? Show your work. (1 mark)

$$y = (-1.31 \times 3.6) + 8.43 \quad | \quad = 3.714 \approx 3.71$$

c. What is the light intensity at the surface? Round your answer to the nearest tenth. Show your work. (1 mark)

$$\begin{aligned} y &= 0 & 0 &= -1.31x + 8.43 & | & x = \frac{8.43}{1.31} \\ & & \Rightarrow 1.31x &= 8.43 & & = 6.4 \text{ lux} \end{aligned}$$

11. Given the equation  $y = -2 \ln x$ , determine all of the characteristics below. (3 marks)

a. x-intercepts

$$(1, 0)$$

b. number of y-intercepts

None

c. the end behavior

It is not a polynomial hence no end behavior.

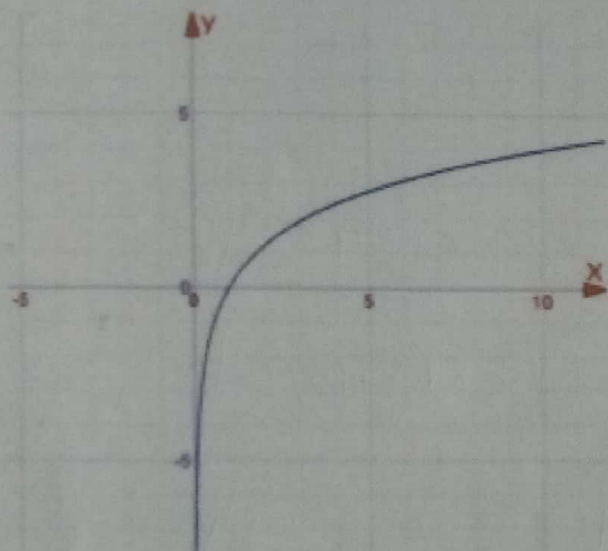
d. domain

$$(0, \infty), \{x | x > 0\}$$

e. range

$$(-\infty, \infty), \{y | y \in \mathbb{R}\}$$

12. State three reasons why you know that the following graph represents a logarithmic function of the form  $y = a \log x$ . (3 marks)



- (i) The point  $(1, 0)$  is on ~~at~~ the graph of all logarithmic functions of the form  $y = a \log x$
- (ii) The graph crosses the x-axis at 1.
- (iii) The curve approaches infinity ~~as~~ with increase in  $x$